A Comparison of Public and Private Partner Selection Models in the Battle of Sexes Game

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***Abstract*—We examine the effect of different partner selection models in population diversity and dynamics when agents interact through a coordination game, namely the Battle of Sexes. In this type of game there are usually more than one Nash Equilibrium. Each one can be considered as a niche that a population may occupy. We compare a partner selection model based on private information with a partner selection based on public opinion. Experimental analysis shows that each one is better than random partner selection, but the outcome is different. While in the private based partner selection usually only one of the Nash Equilibrium survives, in the opinion based partner selection each strategy profile is able to resist. These results were obtained when each strategy was confined to its location and no movement was allowed between places. This raises some questions on how diversity can be maintained with mutation and strategy mobility having a negative impact.**

I. Introduction

In this paper we approach the problem of partner selection from a private and public perspective. Partner selection can be used by interacting agents to select the most favourable one [1], [2]. Partner selection based on private information can be motivated by an agent private knowledge of past interactions [1], [2]. On the other hand partner selection based on public knowledge is similar to a signalling game [3]–[5] where agents only interact if they use the same signal. In games with coordination dilemmas, such signal can be used as coordination device.

When an agent selects partners based on private information, he needs some cognitive capabilities to discern good ones from bad ones. Information from past interactions can be used to decide if a new partner should be seek or not [2], [6], [7]. In such cases there is limited communication between agents on partner quality.

In contrast, when public information is used to select partners, agents can use a common signal [8], use reputation [9], examine their intention [10], or select those with a common norm [5]. In such public schemes one is also interested on how a common norm evolves in a population. Opinion dynamics provides models where one is interested in a evolution of a consensus.

In this paper we compare two methods of partner selection in the context of game theory. One is based on private information collected by a single agent. The other is base on public opinion. In our model, an agent's opinion reflects the way the game should be played. This raises a problem of coordination in that agents that belong to the same strategy profile must reach a common opinion or risk selecting wrong partners. If the game has multiple Nash Equilibria (NEs), then a population composed of such agents can help diversity as each Nash Equilibrium (NE) will have its unique opinion. We also compare the resulting population dynamics with agents that randomly select partners.

II. Related Work

There is research on partner selection [1], [2], [6], [11]–[17]. However, these models are often tailored for a specific game such as Public Good Provision (PGP) or Iterated Prisoner's Dilemma (IPD) [2], [6].

Research similar to ours is [1] and [11] where population structure is able to evolve. Players are embedded in a network. If a player can change his links, selection favours cooperators that prefer to maintain links with their kin and to drop links with defectors. However, their findings were done in 2-player games and they only considered two types of strategies.

Research on opinion dynamics [18]–[20] has largely focused on how a group of agents can reach a consensus in the presence of extremists. There is some research were interactions are mediated by some game [5], although they focus on cooperative games.

When agents select based on public information usually they use some kind of reputation [21], [22]. An agent reputation depends on its past actions which means that the reputation update rule is tied to a particular game. Reputation is also used in online shops where it is built not only from an agent's action in a shop but also from messages or signals that he conveys in order to convince prospective buyers.

III. Model Description

For our model we need an evolutionary algorithm that does not influence any partner selection model. To this end, we use an algorithm without an explicit fitness function, but instead players accumulate energy by playing games, and reproduce when achieve a certain threshold. In this framework, called Energy Based Evolutionary Algorithm (EnBEA), we can apply different partner selection models. Table I presents the parameters discussed in the following sections.

TABLE I. MODEL PARAMETERS RELATED TO AGENTS.

agent chromosome	
s	game strategy
l	pool size
	selection model
δ	probability update factor
π_T	payoff threshold
agent phenotypic traits	
e	energy
a	age
$\mathbf{p}$	probability vector
$\mathbf{c}$	combination vector
o	opinion
u	uncertainty
selection models parameters	
f_{CT}	fraction of combinations transmitted
u_{IF}	uncertainty increase factor
μ	opinion dynamics speed
other parameters	
L	longevity
V	death by old age deviation

A. Population Model

In this section we will give a formal description of EnBEA. It is a population model where agents born, interact, reproduce and die. Agent interaction is mediated by some game. Interaction is essential because agents acquire or loose energy when playing games and energy is necessary to reproduce. Agents can die because of old age, starvation (lack of energy) and overcrowding.

We use games as an energy transfer process. This means a redefinition of the payoff function. A game G is a tuple (N, A, E) where N is a set of n players, $A = \{A_1, \ldots, A_n\}$ and each A_i a set of actions for player i, and $E = \{e_1, \ldots, e_n\}$ is a set of energy functions, $e_i : A_1 \times \ldots \times A_n \to \mathbb{R}$.

The agent parameters related to this model are the strategy s which he uses to play game G, an energy level e and an age a. In each iteration t of EnBEA a population of agents, $\mathcal{P} = \{\alpha_1, \ldots\}$ is updated through three phases.

- play: in this phase all agents play the game and update their energy. Partners can be randomly selected or agents can choose them.
- reproduction: in this phase the agents whose energy is above some threshold produce one offspring and their energy is decremented by some value.
- death: in this phase the entire population goes through a carrying capacity event where the probability to die depends on population size, an old age event where the probability to die depends on agent's age and a check on the agent's energy. Age of surviving agents is incremented by one.

In the play phase, the game is used as energy transfer. Regarding the relation between the payoff function and the energy function, we have extended the approach followed in [7] and considered the case where the obtained energy is scaled and translated to the interval $[-1, 1]$:

$$e \leftarrow e + \frac{\pi}{\max(\overline{\pi}, |\underline{\pi}|)} \quad . \tag{1}$$

where π represents the payoff obtained by an agent, and $\overline{\pi}$ and $\underline{\pi}$ are the highest and lowest payoffs obtainable in game G.

Scaling allows us to compare the evolutionary dynamics of games with different payoff functions, e.g. comparing the number of offspring per iteration or the number of iterations until an extinction occurred. We could remove scaling, if we made energy range equal to payoff range.

With equation (1) we introduce the possibility of an agent dying through starvation when the energy drops below zero, thus augmenting the risk of extinction. Instead of zero, we could have used another energy threshold in the decision to remove agents, but then this would be another parameter in our model. This case is more realistic as the payoff value reflects gains and costs of an agent. Consider for instance, the costs of providing in the Public Good Provision game or of being exploited in the Prisoner's Dilemma game.

When an agent's energy reaches the reproduction threshold e_R, it is decremented by this value, and a new offspring is inserted in the population. Moreover, we have to deal with the possibility of an agent's energy dropping below zero. Similar to [2] we remove an agent when his energy drops below zero. The energy of newborns could be zero, but this puts pressure on the first played games to obtain positive energy, otherwise infancy mortality may be high. Instead we opt for giving newborn e_B units of energy. The dynamics of an agent's energy depends on two parameters, namely e_R and e_B.

In order to avoid exponential growth, in each iteration of the algorithm all agents go through death events. We consider two events: one depends on population size and a second that depends on agent's age. The probability of an agent dying because of population size is:

$$P(\text{death population size}) = \frac{1}{1 + e^{6\frac{K - |\mathcal{P}|}{K}}} \quad , \tag{2}$$

where $|\mathcal{P}|$ is the current population size and K is a parameter that we call carrying capacity. This probability is a sigmoid function. The exponent was chosen because the logistic curve outside the interval $[-6, 6]$ is approximately either zero or one. In the advent of the entire population duplicating size, it will not go from a zero probability of dying to certain extinction. The probability of an agent dying because of old age is:

$$P(\text{death agent's age}) = \frac{1}{1 + e^{\frac{L-a}{V}}} \quad , \tag{3}$$

where L is agents' life expectancy and V controls the deviation at which agents die through old age.

B. Partner Selection Based on Private Information

We choose the model presented in [23]. An agent in this partner selection model has two vectors of size l. Vector $\mathbf{p}$ contains probabilities while vector $\mathbf{c}$ contains sets of candidate partners. When an agent needs to play a game, he selects a set of candidate partners from $\mathbf{c}$. Sets with higher probability have more chance of being picked. After the agent played the game he compares the payoff he obtained with threshold π_T. If the payoff is higher the vectors are not changed. Otherwise, the selected set of candidate partners is replaced by a randomly set and its associated probability is multiplied by factor $\delta < 1$. Since the probability decreases, in order to maintain unit sum, the decreased amount is distributed evenly among the other positions in vector $\mathbf{p}$.

As long as the population remains stable, the net effect of this partner selection is for good sets of candidate partners absorb the probabilities of discarded sets of candidate partners. Whenever an agent that is in a set of candidate partners dies, a randomly set with live candidate partners is inserted in the corresponding vector position.

We augmented this model with the possibility of a parent passing some of his combinations to his offspring. The rationale is to give some information on who are the best partners instead of every newborn having to start from scratch. The combinations that are passed are randomly chosen, without consideration for $\mathbf{p}$. The remaining positions of $\mathbf{p}$ are randomly filled.

C. Partner Selection Based on Public Opinion

We adapted a model of opinion formation [18] to perform partner selection. A player's opinion is a public fact that represents how a game should be played. Players with the same opinion prefer playing among themselves compared to players with opposite opinions. Besides an opinion, a player also has a uncertainty which is used to filter out players with different opinions. Thus a player is characterised by $\alpha = (o, u)$, where o is the opinion and u is the uncertainty.

Opinion dynamics occurs during partner selection and is influenced by available partners and game payoff. Player α_1 starts by selecting the set of candidate partners. A partner α_2 is assigned an weight according to:

$$\begin{cases} 2 - |o_1 - o_2| & \text{if } h_{12} > u_1 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where h_{12} is the overlap between both players' opinion and represents their overall agreement on how to play the game. Following [18], the overlap is defined by:

$$h_{12} = \min(o_1 + u_1, o_2 + u_2) - \max(o_1 - u_1, o_2 - u_2) \tag{5}$$

From equation (4) it can be seen that whenever there is a disagreement on how to play the game, the corresponding partner is not chosen. After computing candidate partners' weight a player performs a weighted random selection of partners. If all candidate partners get zero weight, the player cannot play a game. In this case, the player's uncertainty is increased by factor u_{IF}. The purpose is for a player to become less extreme and accept partners with different opinions.

When $n-1$ partners have been selected, player α_1 plays a game. Afterwards it updates his opinion and uncertainty based on the payoff it obtained. We follow the same procedure described in [18] but we additionally consider the hypothesis of opinions diverging. The rationale is if the payoff is above some threshold, meaning it is considered good, then players' opinions should converge. Otherwise opinions diverge. The opinion and uncertainty update rules are:

$$o_1 \leftarrow o_1 \pm \mu\left(\frac{h_{1\,2}}{u_2} - 1\right)(o_2 - o_1) \tag{6}$$

$$u_1 \leftarrow u_1 \pm \mu\left(\frac{h_{1\,2}}{u_2} - 1\right)(u_2 - u_1) \tag{7}$$

where μ is a parameter that controls the dynamics speed. We add, respectively subtract, if the payoff obtained by player α_i is higher than π_T.

When a parent produces an offspring, he gets the parent's opinion and uncertainty. To reflect the fact that juvenile tend to seek other opinions, his uncertainty is increased by parameter u_{IF}.

D. Battle of Sexes

We have selected the battle of sexes game because it is a coordination dilemma. Two players must decide on which event to go, either a tennis match or an opera concert. If they go to separate events they get zero payoff. Both players are better off if they go together. Each one has his favourite event, the man prefers the tennis match and the woman the opera concert. The player that goes to his favourite event gets a payoff of one. This game as a single parameter, π_U, that is the payoff obtained by the player that does no go to his favourite event when he joins the other partner. The game payoff is thus:

$$\begin{bmatrix} \pi_U, 1 & 0, 0 \\ 0, 0 & 1, \pi_U \end{bmatrix} \tag{8}$$

with the restriction $0 > \pi_U > 1$. The top row and left column correspond to the opera concert, while the bottom row and right column correspond to the tennis match. The man has to select columns while the woman selects rows. In this paper a game strategy s is characterised by the agent's role in the game, either F if he plays the woman or M if he plays the man, and by the agent's action, either 0 if he does not go his favourite event or 1 otherwise. We thus have the following $s \in \{(F, 0), (F, 1), (M, 0), (M, 1)\}$.

IV. Experimental Analysis

We have performed a first set of experiments with a well mixed population and with the two partner selection methods described in previous sections. We have also performed experiments with random partner selection that acted as a control group.

A. Simulation Parameters

We have performed simulations we set up an initial population with two sites. Each site had a carrying capacity of 300. In site one there were ten agents with strategy $(F, 0)$ and ten agents with strategy $(M, 1)$. In the other site there were also twenty agents but with strategies $(F, 1)$ and $(M, 0)$. Interaction between sites was limited to partner selection. When selecting a partner, an agent from the same site and one from the other site have the same probability.

Game payoff π_U was set to 0.5 meaning that whenever both agents selected the same event, the agent that went to his favourite event got twice as much energy as the other agent. Energy needed to reproduce was set to 50 and agent longevity was set to 300. This means that an agent that only selects partners can generate six offspring. Each offspring was subject to a one gene mutation with probability 10%. The offspring was placed in the parent's site.

As for partner selection, we have considered five scenarios:

PCV Agents use the private selection model. All initial agents had the same genes, $l = 4$, $\delta = 0.5$, and $\pi_T = 0.5$. An offspring did not receive any combination from his parent.

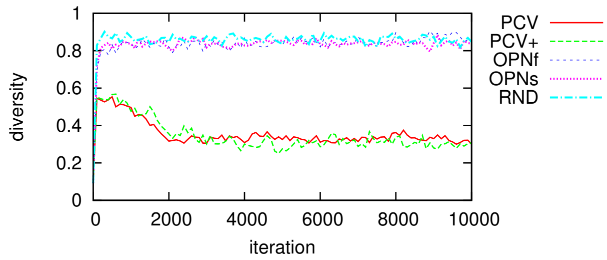


Fig. 1. Agent partners diversity throughout typical simulations in all treatments. In scenarios diversity converges to .3 while in scenarios *PCV*, *PCV+*, *OPNs*, *OPNs* and *RND* diversity converges to .3, .307, .862, .838 and .846, respectively. See electronic version for details.

PCV+ This scenario is equal to *PCV*, but an offspring received half the parent's combinations.

OPNs Agents use the opinion based selection model. The initial opinion and uncertainty were drawn from a normal distribution with standard deviation one. For one site the average was -0.25 while for the other was 0.25. The selection model parameters were $u_{IF} = 1.001$ and $\mu = 0.25$.

OPNf This scenario is equal to *OPNs* but an offspring's uncertainty increased more, $u_{IF} = 1.01$.

RND Agents randomly selected partners.

For all parameter combinations we ran the algorithm for 10000 iterations and performed 30 simulation runs.

B. Simulation Results

The first set of results that we present is the average number of different partners selected by agents. This number gives a rough picture of how selective are the partner selection models and if there are groups of agents that only select among themselves. For each simulation iteration we have considered a window that includes all the selections occurred in past iterations. For every agent in this window we computed how many different partners were selected. We then divide by the number of agents and window size plus one. A value near one means an agent is not selective, while a value near zero means an agent always selects the same partner. Figure 1 shows the evolution in typical simulation runs. The plots were obtained with a window of size ten. In scenarios *PCV* and *PCV+* agents focus on fewer partners than in the other scenarios.

TABLE II. KOLMOGOROV-SMIRNOV STATISTICAL TEST ON AGENT PARTNERS DIVERSITY. ONLY IN SCENARIOS *PCV* AND *PCV+* THE DIVERSITY BELONGS TO THE SAME DISTRIBUTION, MEANING IN THESE SCENARIOS THE DIVERSITY IS SIMILAR.

	$\log_{10}(p\text{-value})$				average diversity
	PCV+	*OPNs*	*OPNf*	*RND*	
PCV	$-.0211$	-12.7	-12.7	-12.7	.402
PCV+		-12.7	-12.7	-12.7	.403
OPNs			-3.88	-10.3	.844
OPNf				-9.49	.84
RND					.862

In order to compare the effect of each partner selection scenario on diversity, for each simulation we computed the average diversity and grouped by scenario, yielding five sets of numbers. We then applied the Kolmogorov-Smirnov test to all pairs of sets to see if they were drawn from the same distribution or not. Table II shows the result of this test and the last column the average diversity. For space considerations we show the base ten logarithm of the p-value. Considering a confidence level of 1% the only scenarios that are similar are the ones using the private partner selection model, *PCV* and *PCV+*. Control scenario *RND* shows the highest diversity since agents randomly select partners, so the chance of picking the same partner decreases as population reaches the carrying capacity. Agents in scenarios *OPNs* and *OPNf* also have a higher diversity mainly because they select partners by their opinion. As the number of available partners with the same opinion increases, so thus partner diversity. Although *OPNs*, *OPNf* and *RND* have high diversity, the p-value is very small. Scenarios *PCV* and *PCV+* have the lowest diversity because of agents' combination vector. Moreover, diversity changes throughout evolution due to the mutations in the pool size gene. In the simulation shown a mutation with lower l appeared around the 500^{th} iteration.

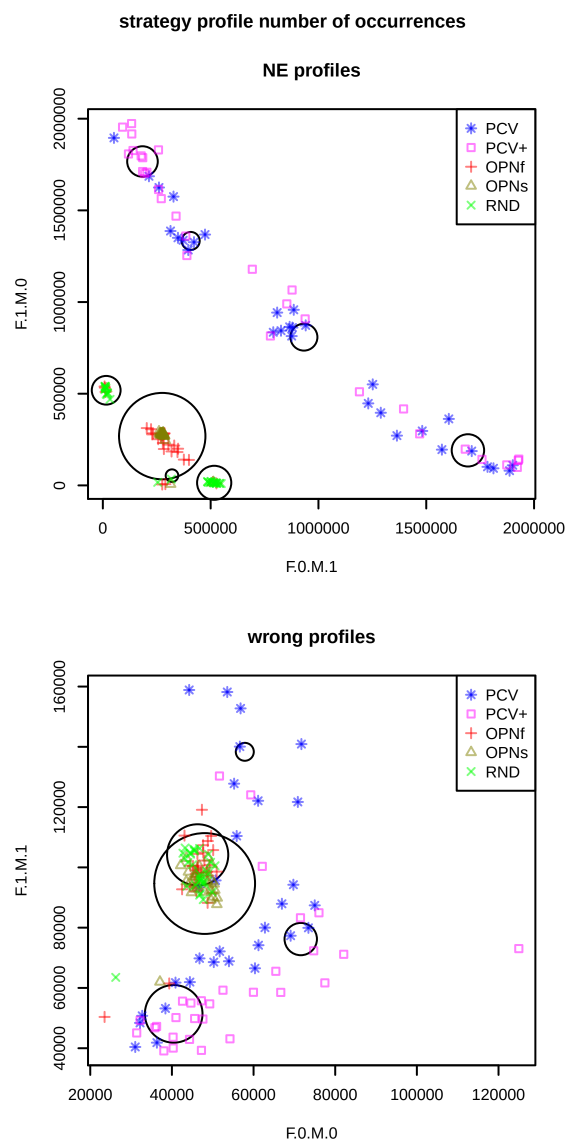


Fig. 2. Number of times a strategy profile has occurred in the scenarios.

The next set of results that we present is the occurrence of specific strategy profiles, presented in figure 2. Each coloured point in this figure represents the data collected during a single run. The black circumferences are the result of applying a Self-Organising Map (SOM) to the points presented in the plot. The size of each circumference represents the number of points mapped to it.

The top plot of figure 2 shows the occurrence of the NE strategy profiles. This is a measure of how successful are agents in selecting the right partners. The horizontal axis represents the number of occurrences of strategy profile $\langle(F,0),(M,1)\rangle$, while the vertical axis stands for strategy profile $\langle(F,1),(M,0)\rangle$. Scenarios *PCV* and *PCV+* show the highest counts due to having the highest number of agents. The SOM circumferences show that in these scenarios either the strategies in both sites are able to prosper or that a single mutant strategy in a site is able to take over. In scenarios *OPNs*, *OPNf* and *RND* there are fewer counts, but population composition is different. While in *RND* strategies converge on a single strategy profile, in *OPNs* and *OPNf* the strategies in both sites are able to thrive.

The bottom plot in figure 2 shows strategies profiles that we call wrong, in the sense that players go separately thus earn zero payoff. The SOM shows a single circumference where all simulations from *OPNs*, *OPNf* and *RND* are located. In this sense opinion based selection is not very different from random partner selection. The *PCV* and *PCV+* scenarios show more dispersion. Overall, wrong strategy profiles occur less often than the NE strategy profiles, with the private based selection model having more success in avoiding wrong strategy profiles. There is a bias towards strategy profile $\langle(F,1),(M,1)\rangle$ because when both NE strategy profiles prevail in the population, strategies $(F,1)$ and $(M,1)$ are the majority in the population, so all things being equal, this strategy profile occurs more often than the other wrong strategy profile.

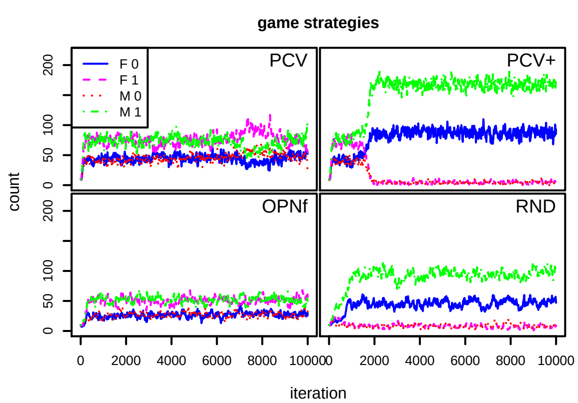
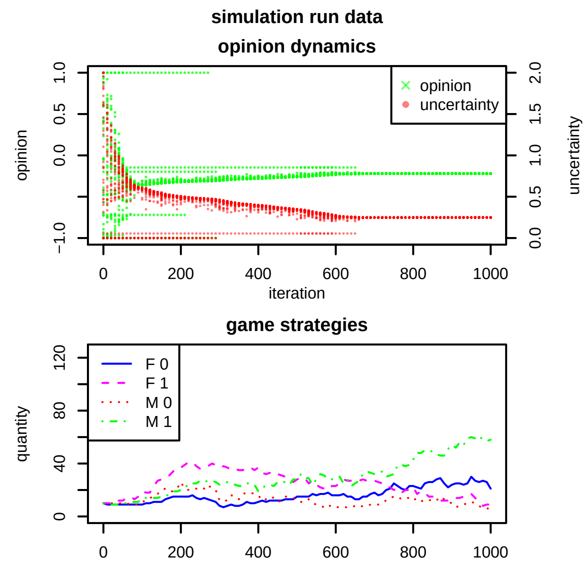
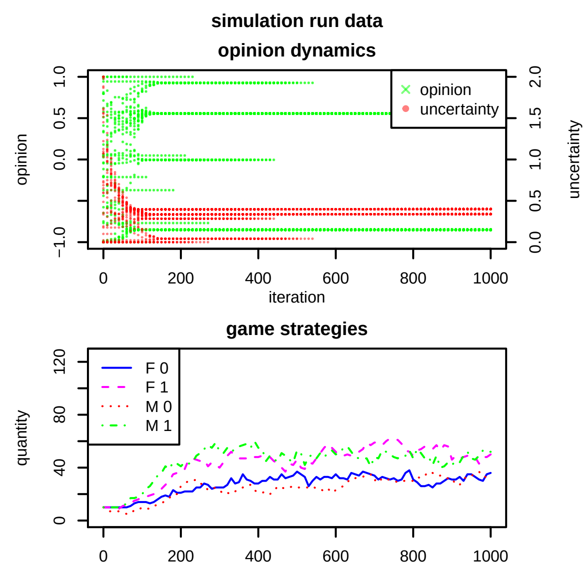


Fig. 3. Population dynamics of typical simulations.

The selection model influences the population dynamics namely if one of the NE strategy profile dominates or both can coexist. Figure 3 shows typical population dynamics for each selection model. In scenarios *PCV* and *PCV+* there is a higher number of agents per iteration. This is a consequence of agents being capable of selecting the right partner, i.e. a male prefers females and vice-versa. In contrast, in scenarios *OPNs*, *OPNf* and *RND* population size is lower. Since in *OPNs* and *OPNf* an agent selects based on a common opinion, males (females) sometimes select other males (females). When this happen, an agent does not earn energy in that iteration. However, in *OPNs* and *OPNf* both NE are able to coexist more often than compared to other scenarios.



(a) Only one strategy profile prevails.



(b) Both strategy profiles are able to coexist for some time.

Fig. 4. Dynamics of samples runs using the opinion selection model. Points taken every 10^{th} iteration.

The last result we discuss is opinion formation. Figure 4 shows two samples of opinion dynamics and games strategies in the first 1000 iterations. We observe that when two opinions fixate in the population (bottom simulation run data plots), then both strategy profiles are able to coexist in the population. The plots only show the first 1000 iterations. From then on population levels and opinion remain the same. In contrast, agent's uncertainty, keeps increasing due to parameter u_{IF}. At some point, there is an overlap between the two niches' opinions and both opinions collapse into a single one. After that one of the two strategy profiles takes over the entire population. This means that as long as two sub-populations have a different opinions that do not overlap (low uncertainty), each sub-population can use its unique strategy.

V. Discussion

We have performed simulations with agents distributed in two niches. Each niche had its own strategy profile. The only interaction between niches was limited to partner selection. Even so, whenever a mutation introduced a new strategy in a niche, this mutant was capable of producing enough offspring to take over both niches. The population performs a random walk with mutation, death by carrying capacity and partner selection dictating which strategy profile prevails. If a parent is capable of putting an offspring in another niche, then such take overs are more frequent and occur sooner. This is not surprising as the fixed points of the underlying system dynamics only contain one of the strategy profiles. A population composed of both strategy profiles is unstable and in the end only one strategy profile will prevail. Even so it is quite remarkable that both private and public based selection models can maintain strategy diversity in the Battle of Sexes game. The opinion based selection with lower uncertainty increase factor is able to sustain a longer diversity when compared with other methods.

VI. Conclusions and Future Work

We have applied two models of partner selection in a coordination game, the *Battle of Sexes*. This game has two NE and we have analysed in which conditions a population initially composed of both NE is able to maintain them. Even when each NE is placed in distinct locations and the only communication between locations is to select partners, a mutant strategy in a location is able to overcome the local strategies and drive the entire population to a single NE. Partner selection models influence such result with a public based selection model showing better resistance to such population homogenisation.

Concerning future extensions of the opinion based selection model, we intend adding to the chromosome the parameters that control opinion dynamics. In this paper, opinions and uncertainties converge (or diverge) at the same speed. If this parameter is turned into a gene, then we can have stubborn agents that never change their opinion or uncertainty, or agents that only change when the payoff is less than some threshold.

We plan comparing these two selection models in games with cooperative dilemma such as Prisoner's Dilemma (PD), PGP or Centipede. If these games are used in the framework of EnBEA there is the chance of a population going extinct because exploiters take over the population or a key role goes extinct. Both private [1], [7] and public models [4], [10] of partner selection have been shown to promote cooperation albeit in fixed-size populations.

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